UNIVERSIDAD NACIONAL DEL CALLAO
FACULTAD DE INGENIERIA ELECTRICA Y ELECTRONICA

PRIMERA PRÁCTICA
TEORIA DE CAMPOS ELECTROMAGNETICOS

2009 - I

1.- Hallar:

a) el gradiente de :

$$H = r^3 \cos\theta \cos\phi$$

(1punto)

b) la divergencia y rotacional de los campos vectoriales:

$$\overline{B} = \rho z^3 \cos\phi\, \hat{\rho} + z^2\, sen^2\phi\, \hat{z}$$

$$\overline{C} = r^2 \cos\theta\, \hat{r} - \frac{1}{r} sen\theta\, \hat{\theta} + 2r^3\, sen\theta\, \hat{\phi}$$

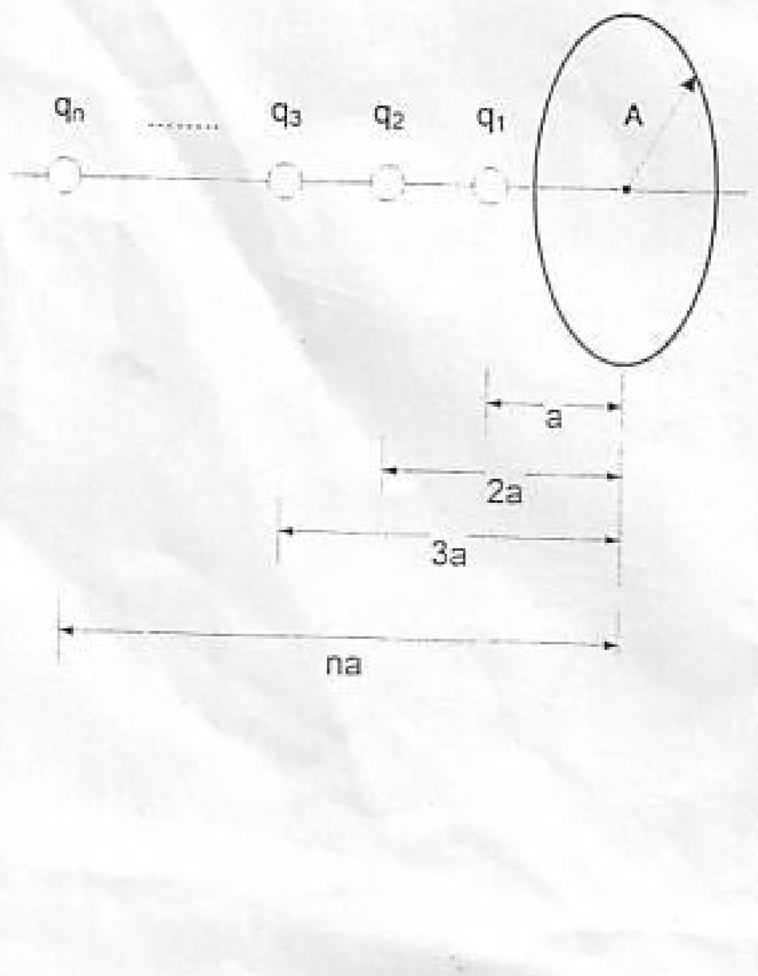
(4 puntos)

2.- El campo eléctrico producido por una esfera con densidad de carga ρ = A/(1+r). radio R para todo el espacio. A=cte

3.-

a) Hallar el flujo, de las n cargas a través del círculo de radio A. (2 p)

b) Hallar la fuerza, entre las n cargas y el círculo con densidad superficial de carga σ. (3 p)



4.- Calcular el potencial y el campo eléctrico en un punto del eje (0,0,z´), debido a un disco de radio R que tiene un espesor e (e << R), y con una densidad de carga ρ constante. Hacer un diagrama de E vs z y U vs z.

Lima 22 de Enero del 2009

1ra práctica de Teoría de Campos Electromagnéticos 22/01/09

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1)

a) $H = r^3 \cos\theta \cos\phi$ (coordenadas esféricas) $\Rightarrow H(r,\theta,\phi)$

$$\bar{\nabla}.H = \frac{\partial H}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial H}{\partial \theta}\hat{\theta} + \frac{\partial H}{\partial \phi}\hat{\phi}$$

$$\bar{\nabla}.H = 3r^2\cos\theta\cos\phi\,\hat{r} + \frac{1}{r}\frac{\partial(r^3\cos\theta\cos\phi)}{\partial\theta}\hat{\theta} + \frac{\partial(r^3\cos\theta\cos\phi)}{\partial\phi}\hat{\phi}$$

$$\bar{\nabla}.H = 3r^2\cos\theta\cos\phi\,\hat{r} + \frac{1}{r}(-r^3\operatorname{sen}\theta\cos\phi)\hat{\theta} + r^3\cos\theta\operatorname{sen}\phi\,\hat{\phi}$$

$$\boxed{\bar{\nabla}.H = 3r^2\cos\theta\cos\phi\,\hat{r} - r^2\operatorname{sen}\theta\cos\phi\,\hat{\theta} - r^3\cos\theta\operatorname{sen}\phi\,\hat{\phi}}$$ 1

b) $\bar{B} = \rho z^3\cos\phi\,\hat{\rho} + z^2\operatorname{sen}^2\phi\,\hat{z}$ (coord. cilíndricas) $\bar{B}(\rho,\phi,z)$

- $$\bar{\nabla}.\bar{B} = \frac{1}{\rho}\frac{\partial(\rho B_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(B_\phi)}{\partial\phi} + \frac{\partial B_z}{\partial z} \quad \left\{\begin{array}{l} B_\rho = \rho z^3\cos\phi \\ B_\phi = 0 \\ B_z = z^2\operatorname{sen}^2\phi\end{array}\right.$$

$$\bar{\nabla}.\bar{B} = \frac{1}{\rho}\frac{\partial(\rho^2 z^3\cos\phi)}{\partial\rho} + \frac{1}{\rho}\frac{\partial(0)}{\partial\phi} + \frac{\partial(z^2\operatorname{sen}^2\phi)}{\partial z}$$

$$\bar{\nabla}.\bar{B} = \frac{1}{\rho}(2\rho z^3\cos\phi) + 0 + 2z\operatorname{sen}^2\phi = 2z^3\cos\phi + 2\operatorname{sen}^2\phi\, z$$ 1

$$\boxed{\bar{\nabla}.\bar{B} = 2z^3\cos\phi + 2\operatorname{sen}^2\phi\, z}$$

- $$\bar{\nabla}\times\bar{B} = \frac{1}{\rho}\begin{vmatrix}\hat{\rho} & \rho\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ B_\rho & \rho B_\phi & B_z\end{vmatrix} = \frac{1}{\rho}\left[\left(\frac{\partial B_z}{\partial\phi} - \frac{\partial(\rho B_\phi)}{\partial z}\right)\hat{\rho} - \left(\frac{\partial B_z}{\partial\rho} - \frac{\partial B_\rho}{\partial z}\right)\rho\hat{\phi} + \left(\frac{\partial(\rho B_\phi)}{\partial\rho} - \frac{\partial B_\rho}{\partial\phi}\right)\hat{z}\right]$$

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$$\bar{\nabla}\times\bar{B} = \frac{1}{\rho}\left[\left(\frac{\partial(z^2\operatorname{sen}^2\phi)}{\partial\phi} - \cancel{\frac{\partial(\rho.0)}{\partial z}}\right)\hat{\rho} - \left(\cancel{\frac{\partial(z^2\operatorname{sen}^2\phi)}{\partial\rho}} - \frac{\partial(\rho z^3\cos\phi)}{\partial z}\right)\rho\hat{\phi} + \left(\cancel{\frac{\partial(\rho.0)}{\partial\rho}} - \frac{\partial(\rho z^3\cos\phi)}{\partial\phi}\right)\hat{z}\right]$$

$$\bar{\nabla}\times\bar{B} = \frac{1}{\rho}\left[2z^2\operatorname{sen}\phi\cos\phi\,\hat{\rho} + 3z^2\rho\cos\phi.\rho\hat{\phi} - (-\rho z^3\operatorname{sen}\phi)\hat{z}\right]$$

$$\bar{\nabla}\times\bar{B} = \frac{2z^2}{\rho}\operatorname{sen}\phi\cos\phi\,\hat{\rho} + 3z^2\rho\cos\phi\,\hat{\phi} + z^3\operatorname{sen}\phi\,\hat{z}$$

$$\boxed{\bar{\nabla}\times\bar{B} = \frac{z^2}{\rho}\operatorname{sen}2\phi\,\hat{\rho} + 3z^2\rho\cos\phi\,\hat{\phi} + z^3\operatorname{sen}\phi\,\hat{z}}$$

$$\bar{C} = r^2 \cos\theta\,\hat{r} - \frac{1}{r}\sin\theta\,\hat{\theta} + 2r^3 \sin\theta\,\hat{\phi} \quad \text{(coord. esféricas)}$$

- $$\bar{\nabla}\cdot\bar{C} = \frac{1}{r^2}\frac{\partial(r^2 C_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(B_\theta \sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(B_\phi)}{\partial\phi}$$

$$\bar{\nabla}\cdot\bar{C} = \frac{1}{r^2}\frac{\partial(r^2 r^2\cos\theta)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(-\frac{1}{r}\sin\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(2r^3\sin\theta)}{\partial\phi}$$

$$\bar{\nabla}\cdot\bar{C} = \frac{1}{r^2}(4r^3\cos\theta) + \frac{1}{r\cancel{\sin\theta}}\left(-\frac{1}{r}\cdot 2\cancel{\sin\theta}\cos\theta\right) + \frac{1}{r\sin\theta}(0)$$

$$\bar{\nabla}\cdot\bar{C} = 4r\cos\theta - \frac{2\cos\theta}{r^2} = 2\cos\theta\left[2r - \frac{1}{r^2}\right]$$

$$\boxed{\bar{\nabla}\cdot\bar{C} = 2\cos\theta\left[2r - \frac{1}{r^2}\right]}$$

- $$\bar{\nabla}\times\bar{C} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ B_r & rB_\theta & r\sin\theta\,B_\phi \end{vmatrix}$$

$$\bar{\nabla}\times\bar{C} = \frac{1}{r^2\sin\theta}\left[\left(\frac{\partial(r\sin\theta B_\phi)}{\partial\theta} - \frac{\partial(rB_\theta)}{\partial\phi}\right)\hat{r} - \left(\frac{\partial(r\sin\theta B_\phi)}{\partial r} - \frac{\partial(B_r)}{\partial\phi}\right)r\hat{\theta} + \left(\frac{\partial(rB_\theta)}{\partial r} - \frac{\partial(B_r)}{\partial\theta}\right)r\sin\theta\,\hat{\phi}\right]$$

$$\bar{\nabla}\times\bar{C} = \frac{1}{r^2\sin\theta}\left[\left(\frac{\partial(r\sin\theta\cdot 2r^3\sin\theta)}{\partial\theta} - \frac{\partial(r\cdot -\frac{1}{r}\sin\theta)}{\partial\theta}\right)\hat{r} - \left(\frac{\partial(r\sin\theta\cdot 2r^3\sin\theta)}{\partial r} - \frac{\partial(r^2\cos\theta)}{\partial\phi}\right)r\hat{\theta} + \left(\frac{\partial(r\cdot -\frac{1}{r}\sin\theta)}{\partial r} - \frac{\partial(r^2\cos\theta)}{\partial\theta}\right)r\sin\theta\,\hat{\phi}\right]$$

$$\bar{\nabla}\times\bar{C} = \frac{1}{r^2\sin\theta}\left[2r^4(2\sin\theta\cos\theta) + \cos\theta\right]\hat{r} - \frac{1}{r^2\sin\theta}\left[8r^3\sin^2\theta - 0\right]r\hat{\theta} + \frac{1}{r^2\sin\theta}\left[0 - -r^2\sin\theta\right]r\sin\theta\,\hat{\phi}$$

$$\bar{\nabla}\times\bar{C} = \frac{1}{r^2\sin\theta}(2r^4\sin 2\theta)\,\hat{r} - \frac{1}{r^2\sin\theta}\cdot 8r^3\sin^2\theta\cdot r\hat{\theta} + \frac{1}{r^2\sin\theta}\cdot r^2\sin\theta\, r\sin\theta\,\hat{\phi}$$

$$\cancel{\bar{\nabla}\times\bar{C} = 4r^2\cos\theta\,\hat{r}\cos\theta}$$

$$\bar{\nabla}\times\bar{C} = \left(\frac{4r^2\sin\theta\cos\theta}{r^2\sin\theta} + \frac{\cos\theta}{r^2\sin\theta}\right)\hat{r} - \frac{8r^4\sin^2\theta}{r^2\sin\theta}\,\hat{\theta} + \frac{r^3\sin^2\theta}{r^2\sin\theta}\,\hat{\phi}$$

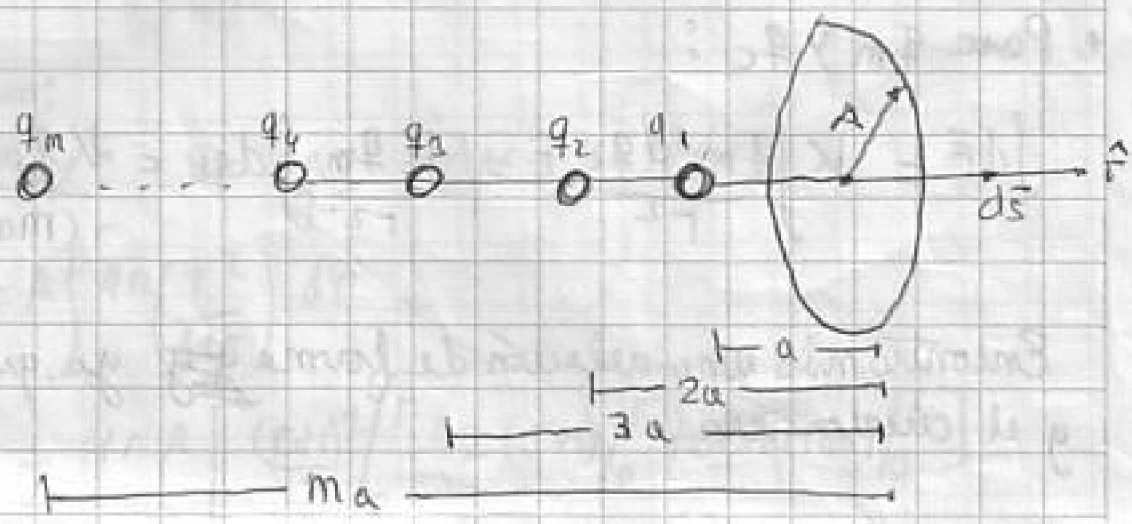
$$\boxed{\bar{\nabla} \times \bar{C} = \left(4\cos\theta + \frac{\cot\theta}{r^2}\right)\hat{r} - 8r^2 \operatorname{sen}\theta\ \hat{\theta} + r\operatorname{sen}\theta\ \hat{\phi}}$$

3)

a) Por ley de Gauss:

flujo Eléctrico:

$$\int d\phi = \oint_S \bar{E}\cdot d\bar{s} = \oint_S E\hat{r}\,ds\hat{r} = \int E\,ds$$



+ Para la primera carga "q_1":

$$\phi_1 = E\int_S ds$$

$$\boxed{\phi_1 = E\cdot S}$$

donde: $\vec{E} = \frac{q_1}{4\pi r^2 \varepsilon_0}\hat{r}$

r = distancia al circulo desde cada esfera. ; $S = \pi A^2$ (Área del circulo donde se halla el flujo total)

$$\Rightarrow \phi_1 = \frac{q_1}{4\pi a^2\varepsilon_0}\cdot \pi A^2$$

+ Para la segunda carga "q_2":

$$\phi_2 = E\int_S ds$$

$$\phi_2 = E\cdot S$$

$$\phi_2 = \frac{q_2}{4\pi(2a)^2\varepsilon_0} = \frac{q_2}{4\pi(2a)^2\varepsilon_0}$$

+ Para la tercera carga "q_3":

$$\phi_3 = E\cdot\int ds$$

$$\phi_3 = \frac{q_3}{4\pi(3a)^2\varepsilon_0} = \frac{q_3}{4\pi(3a)^2\varepsilon_0}$$

$\Rightarrow$ haciendo una forma para hallar el flujo total y analizando:

$$\boxed{\phi_T = \phi_1 + \phi_2 + \phi_3 + \dots\ \phi_m}$$

$$\therefore \phi_T = \frac{q_1}{4\pi a^2\varepsilon_0} + \frac{q_2}{4\pi(2a)^2\varepsilon_0} + \frac{q_3}{4\pi(3a)^2\varepsilon_0} + \dots + \frac{q_m}{4\pi(ma)^2\varepsilon_0}$$

Hallando una progresión:

$$\phi_T = \frac{1}{4\pi\varepsilon_0 a^2}\left[q_1 + \frac{q_2}{2^2} + \frac{q_3}{3^2} + \frac{q_4}{4^2} + \dots + \frac{q_m}{m^2}\right]$$

$$\phi_T = \frac{1}{4\pi\varepsilon_0 a^2}\sum_{i=1}^{m}\left(\frac{q_i}{i^2}\right)$$

$$\Rightarrow \boxed{\phi_T = \frac{1}{4\pi a^2\varepsilon_0}\sum_{i=1}^{m}\frac{q_i}{i^2}}$$

6) Fuerza entre cargas discretas:

$$d\vec{F} = K\int \frac{q\,dq}{r^2}\,\hat{r}$$

* Para q_1 y q_c:

$$\int d\vec{F} = K\int \frac{q_1\,dq_c}{r^2}\hat{r} = \frac{Kq_1}{r^2}\int dq_c = \frac{Kq_1 q_c}{a^2} = \frac{Kq_1(\sigma\pi A^2)}{a^2} = \vec{F}_{1c}$$

* Para q_2 y q_c:

$$\int d\vec{F} = K\int \frac{q_2\,dq_c}{r^2}\hat{r} = \frac{Kq_2}{r^2}\int dq_c = \frac{Kq_2 q_c}{(2a)^2} = \frac{Kq_2(\sigma\pi A^2)}{(2a)^2} = \vec{F}_{2c}$$

* Para q_m y q_c:

$$\int d\vec{F} = K\int \frac{q_m\,dq_c}{r^2}\hat{r} = \frac{Kq_m}{r^2}\int dq_c = \frac{Kq_m q_c}{(ma)^2} = \frac{Kq_m(\sigma\pi A^2)}{(ma)^2}\hat{r} = \vec{F}_{mc}$$

Encontramos una relación de forma ~~ya~~ ya que la fuerza de los "m" cargas y el círculo sera:

$$F_T = F_{1c} + F_{2c} + F_{3c} + \dots\ F_{mc}$$

$$F_T = K\int \frac{q_1\,dq_c}{r^2} + K\int \frac{q_2\,dq_c}{r^2} + \dots\ K\int \frac{q_m\,dq_c}{r^2}$$

Reemplazando datos:

$$F_T = \frac{Kq_1(\sigma\pi A^2)}{a^2} + \frac{Kq_2(\sigma\pi A^2)}{(2a)^2} + \dots\ \frac{Kq_m(\sigma\pi A^2)}{(am)^2}$$

$$\Rightarrow F_T = \frac{\sigma\pi A^2}{4\pi\varepsilon_0 a^2}\left[\frac{q_1}{1} + \frac{q_2}{2^2} + \frac{q_3}{3^2} + \dots\ \frac{q_m}{m^2}\right]$$

$$\therefore\ \boxed{F_T = \frac{\sigma\pi A^2}{4\pi\varepsilon_0 a^2}\sum_{i=1}^{m}\frac{q_i}{i^2}}$$

A = radio círculo
a = distancia
σ = densidad de carga

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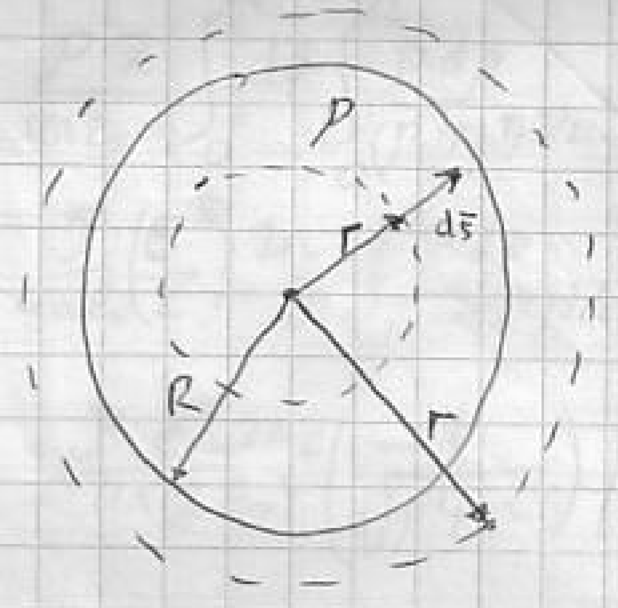
2)



Hallando campo para: $r<R$

a) Usando Ley de Gauss:

$$\oint_S \bar{E}\cdot d\bar{s} = \frac{q}{\varepsilon_0}$$

$$\oint_S E\hat{r}\cdot ds\,\hat{r} = \frac{q}{\varepsilon_0}$$

$$E\cdot ds = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0} \ldots (1)$$

hallando la carga encerrada en la región para $r<R$

$dq = \rho\, dv$ integrando:

$$\boxed{\int_0^q dq = \int \rho\, dv}$$ donde: $V = \frac{4\pi r^3}{3}$; $dV = 4\pi r^2 dr$

$$q = \int \left(\frac{A}{1+r}\right)\cdot 4\pi r^2 dr = A\int_0^r 4\pi\left(\frac{r^2}{1+r}\right) dr$$

$$q = 4\pi A\int_0^r \left(\frac{r^2}{1+r}\right) dr = 4\pi A\left[\frac{(r+1)^2}{2}\Big|_0^r - 2(r+1)\Big|_0^r + \ln(r+1)\Big|_0^r\right]$$

$$q = 4\pi A \quad \cancel{\frac{(r+1)^2}{2} - 2(r+1)}$$

$$q = 4\pi A\left[\frac{(r+1)^2}{2} - \frac{1}{2} - 2(r+1) + 2 + \ln(r+1)\right]$$

$$q = 4\pi A\left[\frac{r^2 - 2r - 4 + 2\ln(r+1)}{2}\right] \ldots (2)$$

Reemplazando (2) en (1):

$$E = \frac{\cancel{4\pi} A\left[r^2 - 2r - 4 + 2\ln(r+1)\right]}{2\cdot \cancel{4\pi} r^2 \varepsilon_0} = \frac{A\left[r^2 - 2r - 4 + 2\ln(r+1)\right]}{2r^2\varepsilon_0}$$

2.5

b) Usando ley de Gauss: $r>R$

$$\oint E\cdot d\bar{s} = \frac{q'}{\varepsilon_0}$$

$$E\cdot(4\pi r^2) = \frac{q'}{\varepsilon_0}$$

donde: $\cancel{dq' = \rho\, dV}$

$$\cancel{\int dq' = \int_0^R \left(\frac{A}{1+r}\right)\cdot 4\pi r^2 dr}$$

$$\cancel{q' = A4\pi\int_0^R \left(\frac{r^2}{1+r}\right) dr}$$

$$q' = \cancel{4\pi A}$$

$$q' = 4\pi A$$

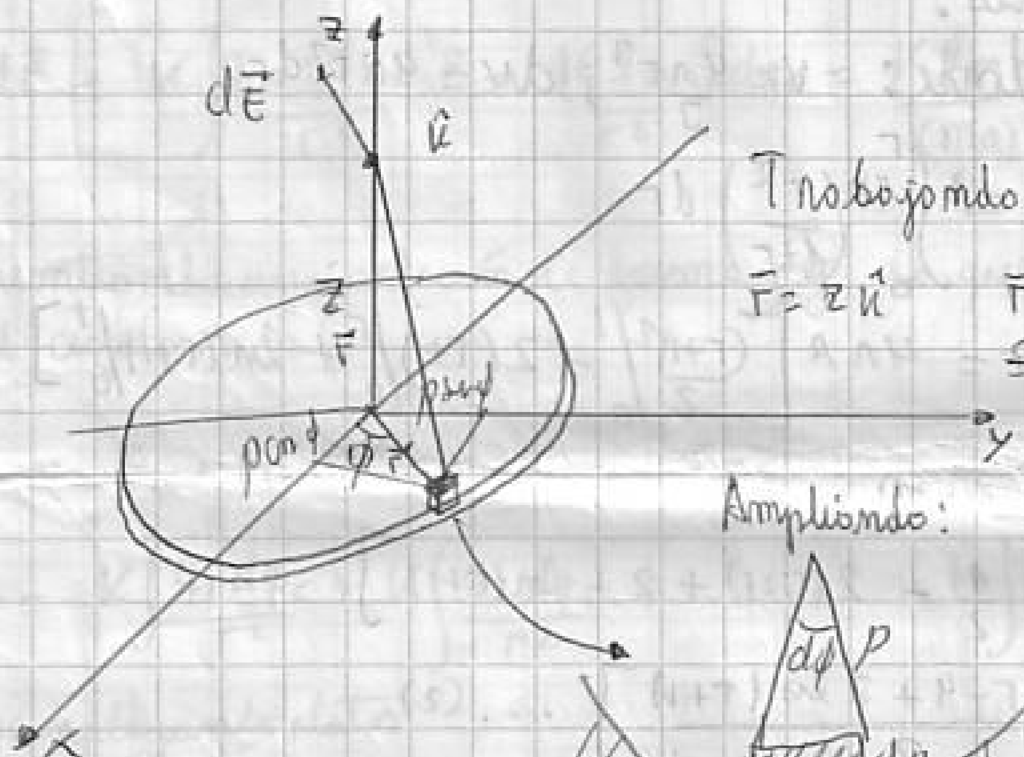
$$E = \frac{q'}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{\rho\cdot V}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{\left(\frac{A}{1+r}\right)\cdot\frac{4\pi R^3}{3}}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{A\, R^3}{3(1+r)\varepsilon_0 r^2}$$

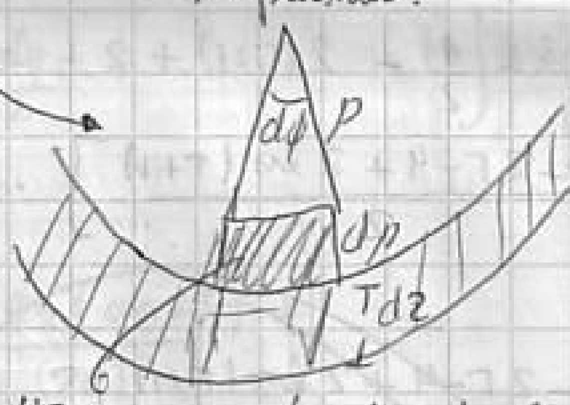
4)



Trabajando en coord. cilíndricas:

$\vec{r} = z\hat{k}$ $\quad \vec{r}\,' = (\rho\cos\phi, \rho\sin\phi, 0)$

Ampliando:



$$dV = \rho\, d\rho\, d\phi\, dz\, (\hat{k})$$

Hallando el $\vec{E}$:

$$\vec{E} = \frac{1}{4\pi\varepsilon}\int_V \frac{dq'(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \qquad dq' = \rho\, dV' \qquad \vec{r}\,' = (\rho\cos\phi, \rho\sin\phi, 0)$$

ρ = densidad de carga $\qquad \vec{r} = z\hat{k}$

$$\vec{r}-\vec{r}\,' = (-\rho\cos\phi, -\rho\sin\phi, z)$$

$$|\vec{r}-\vec{r}\,'| = \sqrt{\rho^2+z^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho\, dv'(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

Reemplazando:

$$\vec{E} = \frac{\rho}{4\pi\varepsilon_0}\iiint \frac{(-\rho\cos\phi, -\rho\sin\phi, z)(dV)}{(\rho^2+z^2)^{3/2}} = \frac{\rho}{4\pi\varepsilon_0}\int \frac{(-\rho\cos\phi, -\rho\sin\phi, z)\,\rho\, d\rho\, d\phi\, dz\, \hat{u}}{(\rho^2+z^2)^{3/2}}$$

$$\vec{E} = \frac{\rho}{4\pi\varepsilon_0}\iiint \frac{z\cdot \rho\, d\rho\, d\phi\, dz}{(\rho^2+z^2)^{3/2}}\,\hat{u}$$

$$\bar{E}= \frac{\mathcal{P}\, Z}{4\pi\varepsilon_0}\int_0^{2\pi} d\phi \int_0^{R} \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}} \int_0^{e} dz\ \hat{k} =$$

$$\bar{E}= \frac{\mathcal{P}}{4\pi\varepsilon_0}\left(\frac{e}{z}\right) 2\pi \int_0^{R} \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\ \hat{k}$$

$$\bar{E}= \frac{\mathcal{P}\, e\, 2\pi z}{4\pi(2\varepsilon_0)}\left(\frac{-1}{\sqrt{\rho^2+z^2}}\right)\Bigg|_0^{R} = \frac{\mathcal{P} e\, 2\pi z}{4\pi 2\varepsilon_0}\left(-\frac{1}{\sqrt{R^2+z^2}}+\frac{1}{z}\right)\hat{k}$$

$$\bar{E}_{(\bar{r})}= \frac{\mathcal{P}\, e\, z}{2\varepsilon_0}\left[\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right]\hat{k} = \frac{\mathcal{P}\, e}{2\varepsilon_0}\left[\frac{z}{|z|}-\frac{z}{\sqrt{R^2+z^2}}\right]\hat{k}$$

$\mathcal{P}$ = densidad constante.

Hallando el potencial:

$$U_{(\bar{r})}= \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq'}{|\bar{r}-\bar{r}'|}$$

$dq' = \mathcal{P}\, dv$
$dv = \rho\, d\rho\, d\phi\, dz$

$$U_{(\bar{r})}= \frac{1}{4\pi\varepsilon_0}\iiint \frac{\mathcal{P}\cdot \rho\, d\rho\, d\phi\, dz}{(\rho^2+z^2)^{3/2}}$$

$$U_{(\bar{r})}= \frac{\mathcal{P}}{4\pi\varepsilon_0}\iiint \frac{\rho\, d\rho\, d\phi\, dz}{(\rho^2+z^2)^{3/2}}$$

$$U_{(\bar{r})}= \frac{\mathcal{P}}{4\pi\varepsilon_0}\cdot\int_0^{2\pi} d\phi \cdot \int_0^{e} dz \cdot \int_0^{R} \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}\ \hat{k}$$

$$U_{(\bar{r})}= \frac{\mathcal{P}}{4\pi\varepsilon_0}\, 2\pi e\left[-\frac{1}{(\rho^2+z^2)^{1/2}}\right]\Bigg|_0^{R}\hat{k}$$

$$U_{(\bar{r})}= \frac{\mathcal{P}\cdot e}{2\varepsilon_0}\left[-\frac{1}{\sqrt{R^2+z^2}}+\frac{1}{z}\right]\hat{k}$$

$$U_{(\bar{r})}= \frac{\mathcal{P}\cdot e}{2\varepsilon_0}\left[\frac{1}{z}-\frac{1}{\sqrt{R^2+z^2}}\right]\hat{k}$$

B