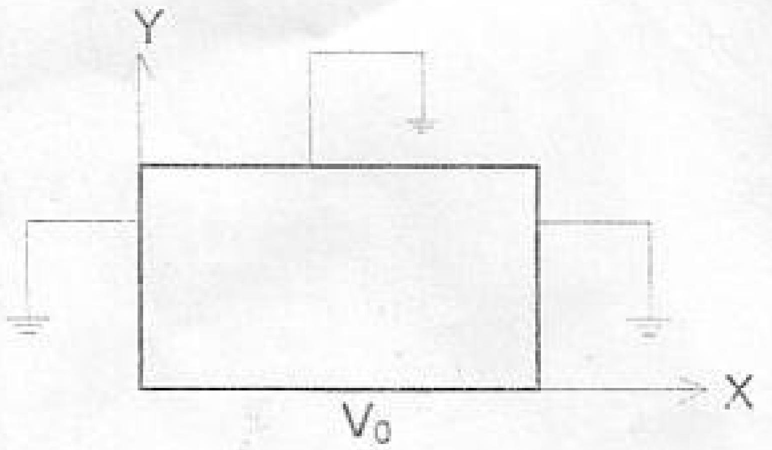
# Examen Parcial

1.- Hallar el potencial electrostático $V(x,y)$, usando la ecuación de Laplace, en el siguiente sistema bidimensional que se muestra en la figura 1 *(5 puntos)*.



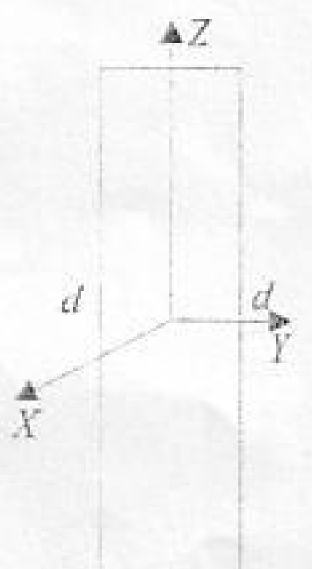

2.- Dada la distribución de carga en el entorno del plano XY, determinada por la siguiente densidad.

$$\rho = \begin{cases} \rho_0 \left(\frac{y}{d}\right) & para\ |y| \leq d \\ 0 & para\ |y| \geq d \end{cases}$$

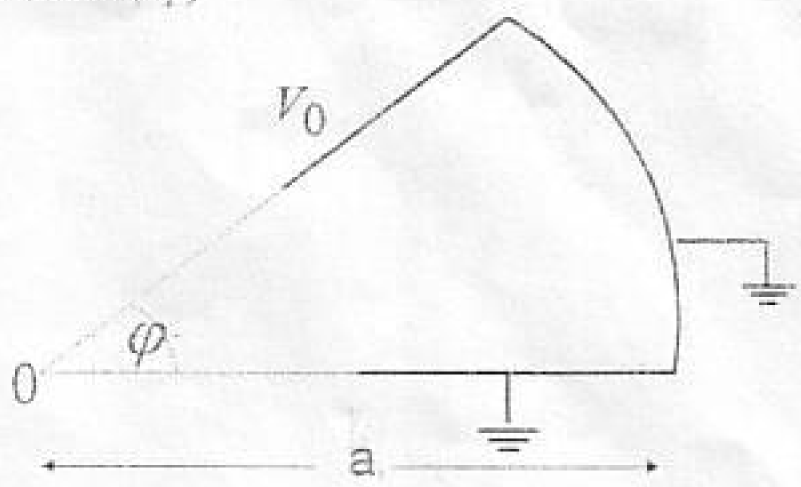
Calcular:

a) el campo eléctrico en cualquier punto del eje Y. (3p)

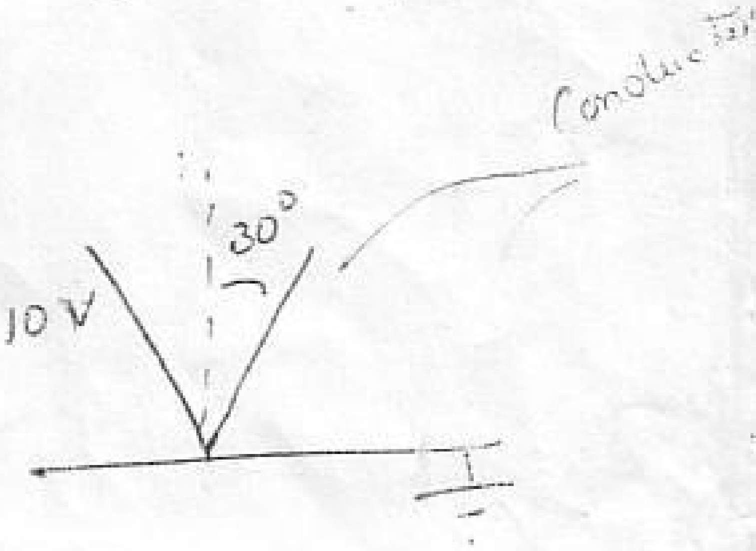
b) el potencial electrostático en la zona $-d \leq y \leq d$. Tomando como referencia el potencial en $y = -d$. (2p).




3.- Una tubería infinita de sección transversal es un sector circular, cuyas potenciales se indican en la figura, se pide hallar el potencial electrostático dentro del recinto *(5p)*




4)




hallar. V, E y σ en la región entre los 2 conductores.

Examen Parcial de Teoría de campos Electromagnéticos   05/02/09

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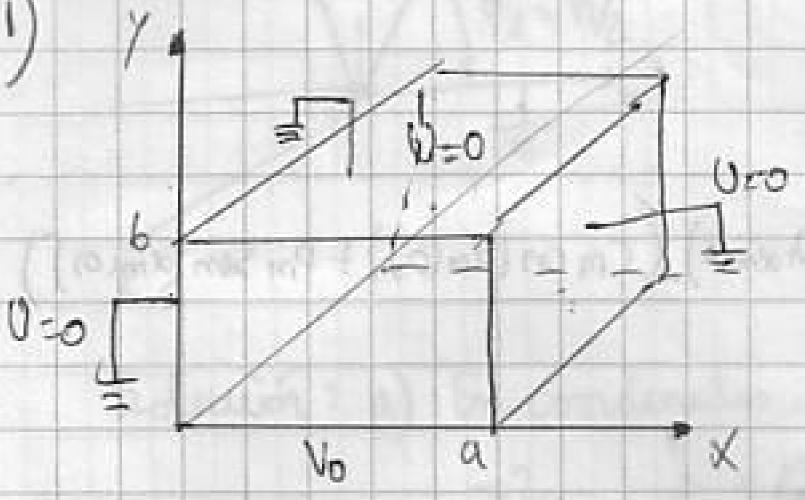
1)




Solución: $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0$ (Ecuación de Laplace)

Como el potencial no depende de Z:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \ldots (1) \quad V_{(x,y)} = X_{(x)} \cdot Y_{(y)} \ldots (2)$$

(2) en (1): $YX'' + XY'' = 0$ (dividiendo entre $XY$):

$$\underbrace{\frac{1}{X}\frac{\partial^2 X}{\partial x^2}}_{\alpha^2} + \underbrace{\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}}_{-\alpha^2} = 0 \ldots (3)$$

Tomando la ecuación que se generó de la ecuación (3):

$$V(x,y) = \sum_{m=1}^{\infty} (A_m \cos h\alpha_m x + B_m \operatorname{Sen} h\alpha_m x) \cdot (C_m \cos(\alpha_m y) + D_m \operatorname{Sen}(\alpha_m y)) \ldots (4)$$

Definiendo condiciones de frontera

$$\begin{cases} X=0 \; ; \; V=0 & 1^{ra} \\ X=a \; ; \; V=0 & 2^{da} \\ Y=b \; ; \; V=0 & 3^{ra} \\ Y=0 \; ; \; V=V_0 & 4^{ta} \end{cases}$$

Para: $X=0$ ; $V=0$ Reemplazando en (4):

$1^{ra}$ C.F $\quad 0 = \sum_{m=1}^{\infty} (A_m \cos h(\alpha_m 0) + B_m \operatorname{Sen} \cancel{h(\alpha_m(0))}) (C_m \cos(\alpha_m y) + D_m \operatorname{Sen}(\alpha_m y))$

$$0 = \sum_{m=1}^{\infty} (A_m)(C_m \cos(\alpha_m y) + D_m \operatorname{Sen}(\alpha_m y))$$

$\boxed{A_m = 0}$ .... (5)

Para: $X=a$ ; $V=0$ Reemplazando en (4):

$2^{da}$ C.F $\quad 0 = \sum_{m=1}^{\infty} (\cancel{A_m} \cos h(\alpha_m a) + B_m \operatorname{Sen} h(\alpha_m a))(C_m \cos(\alpha_m y) + D_m \operatorname{Sen}(\alpha_m y))$

$$0 = \sum_{m=1}^{\infty} (B_m \operatorname{Sen} h(\alpha_m a)) \cdot (C_m \cos(\alpha_m y) + D_m \operatorname{Sen}(\alpha_m y))$$

$0 = B_m \operatorname{Sen} h(\alpha_m a) \Rightarrow m\pi = \alpha_m a \quad \boxed{\alpha_m = \frac{m\pi}{a}} \ldots (5)$

Para: $y=b$ ; $U=0$

3ra C.F

$$0=\sum_{m=1}^{\infty}\left(\cancel{A_m \cosh \alpha_m x}+B_m \operatorname{senh}\alpha_m x\right)\left(C_m \cos(\alpha_m y)+D_m \operatorname{sen}(\alpha_m y)\right)$$

$$0=\left(B_m \operatorname{senh}\alpha_m x\right)\left(C_m \cos(\alpha_m b)+D_m \operatorname{sen}(\alpha_m b)\right) \ldots (6)$$

Para: $y=0$ ; $U=V_0$

4ta C.F

$$V_0=\sum_{m=1}^{\infty}\left(\cancel{A_m \cosh \alpha_m x}+B_m \operatorname{senh}\alpha_m x\right)\cdot\left(C_m \cos(\alpha_m(0))+D_m \operatorname{sen}(\alpha_m(0))\right)$$

$$V_0=\sum_{m=1}^{\infty}\underbrace{C_m B_m}_{E_m} \operatorname{senh}(\alpha_m x)$$

Usando Fourier: $0<x<a$

$$E_m=\frac{2}{T}\int_0^T V_0 \operatorname{senh}(\alpha_m x)\,dx$$

$$E_m=\frac{2}{a}\int_0^a V_0 \operatorname{senh}(\alpha_m x)\,dx$$

$$E_m=\frac{2V_0}{a}\cdot\frac{\cosh(\alpha_m x)}{\alpha_m}\Big|_0^a=\frac{2V_0}{\cancel{a}\left(\frac{m\pi}{\cancel{a}}\right)}\left[\cosh(\alpha_m x)\Big|_0^a\right]$$

$$E_m=\frac{2V_0}{m\pi}\left[\cosh\left(\cancel{a}\frac{m\pi}{\cancel{a}}\right)-\cosh(\alpha_m(0))\right]=$$

$$E_m=\frac{2V_0}{m\pi}\left[\cosh(m\pi)-1\right] \ldots (A)$$

De (6) en (5) $\Rightarrow$ $0=C_m \cos(\alpha_m b)+D_m \operatorname{sen}(\alpha_m b)$

$$C_m \cos(\alpha_m b)=-D_m \operatorname{sen}(\alpha_m b)$$

$$D_m=\cos(\alpha_m b) \wedge C_m=-\operatorname{sen}(\alpha_m b) \quad \alpha_m=\frac{m\pi}{a}$$

$$E_m=C_m B_m$$

$$B_m=\frac{E_m}{C_m}=\frac{\frac{2V_0}{m\pi}\left[\cosh(m\pi)-1\right]}{-\operatorname{sen}\left(\frac{m\pi b}{a}\right)}$$

$$\boxed{B_m=\frac{2V_0}{m\pi}\left[\frac{1-\cosh(m\pi)}{\operatorname{sen}\left(\frac{m\pi b}{a}\right)}\right]} \wedge \boxed{C_m=-\operatorname{sen}\left(\frac{m\pi b}{a}\right)}$$

$$\boxed{D_m=\cos\left(\frac{m\pi b}{a}\right)}$$

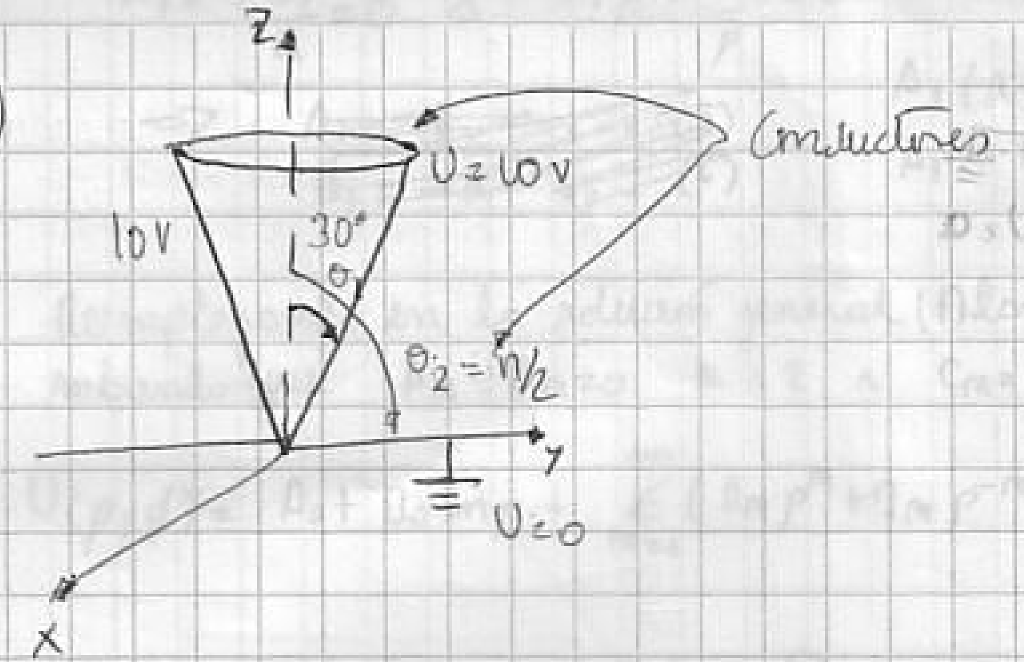
$\Rightarrow$ Reemplazando en la ecuación general:

$$U(x,y)=\sum_{m=1}^{\infty}\left(B_m \operatorname{senh}\alpha_m x\right)\left(\cancel{C_m}\cos(\alpha_m y)+D_m \operatorname{sen}(\alpha_m y)\right)$$

$$U(x,y)=\sum_{m=1}^{\infty}\left(\frac{2V_0}{m\pi}\left(\frac{1-\cosh(m\pi)}{\operatorname{sen}\left(\frac{m\pi b}{a}\right)}\right)\right)\operatorname{sen}\left(\frac{m\pi x}{a}\right)\cdot\left[-\operatorname{sen}\left(\frac{m\pi b}{a}\right)\cos\left(\frac{m\pi}{a}y\right)+\cos\left(\frac{m\pi b}{a}\right)\operatorname{sen}\left(\frac{m\pi}{a}y\right)\right]$$

$$\therefore U(x,y)=\sum_{m=1}^{\infty}\frac{2V_0}{m\pi}\operatorname{sen}\left(\frac{m\pi}{a}\right)x\cdot\left(\frac{1-\cosh m\pi}{\operatorname{sen}\left(\frac{m\pi b}{a}\right)}\right)\cdot\left[-\operatorname{sen}\left(\frac{m\pi b}{a}-\frac{m\pi y}{a}\right)\right]$$

4)




Solución: a) En coordenadas esféricas el potencial no depende de $r$ y $\phi$:

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\cancel{\left(r^2\frac{\partial V}{\partial r}\right)} + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial V}{\partial\theta}\right) + \cancel{\frac{1}{r^2\operatorname{sen}^2\theta}\frac{\partial^2 V}{\partial\phi^2}} = 0$$

Si depende de $\theta$

donde: $\frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial\theta}\left(\operatorname{sen}\theta\frac{\partial V}{\partial\theta}\right)=0$

$$\frac{\partial V}{\partial\theta} = \frac{C_1}{\operatorname{sen}\theta}$$

$$\partial V = \frac{C_1\, d\theta}{\operatorname{sen}\theta}$$

$$V_{(\theta)} = C_1 \ln\left(\operatorname{tg}\left(\tfrac{\theta}{2}\right)\right) + C_2 \quad \ldots (1)$$

Usando las condiciones de frontera:

$V(\theta_1 = 30°) = 10$ En (1): $10 = C_1 \ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right) + C_2$

$V(\theta_2 = \pi/2) = 0$ $\quad 0 = C_1 \ln\left(\operatorname{tg}\left(\frac{\pi}{4}\right)\right) + C_2$

$$10 = C_1 \ln\left(\operatorname{tg}\left(\tfrac{\pi}{12}\right)\right)$$

$$C_1 = \frac{10}{\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)} \quad ; C_2 = 0$$

falta operar

Reemplazando en (1): $V(\theta) = \frac{10}{\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)} \ln\left(\operatorname{tg}\left(\frac{\theta}{2}\right)\right)$

3.5

b) $$\bar{E} = -\nabla V = -\frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta} = -\frac{1}{r}\left[\frac{10}{\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)}\right]\frac{1}{2}\frac{\sec^2\left(\frac{\theta}{2}\right)\hat{\theta}}{\operatorname{tg}\left(\frac{\theta}{2}\right)} = -\frac{10}{r\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)}\cdot\frac{1}{2\operatorname{sen}\frac{\theta}{2}\cos\frac{\theta}{2}}\hat{\theta}$$

$$\boxed{\bar{E} = -\frac{10}{r\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)\operatorname{sen}\theta}\hat{\theta}}$$

?

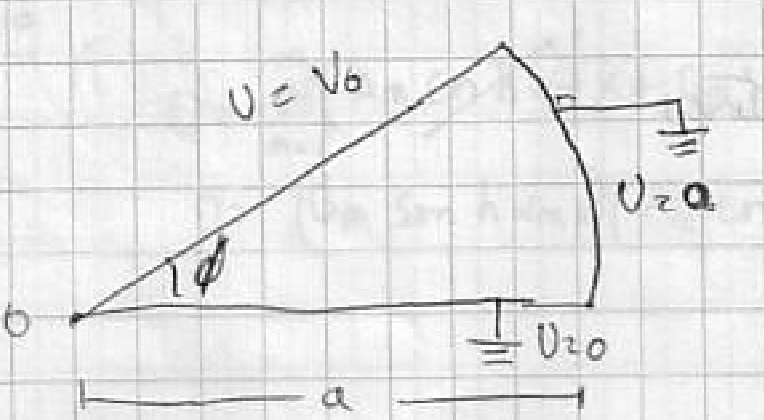
c) hallando las densidades de carga de cada conductor en la región de ellos:

$$\sigma/_{\theta=\pi/6} = \varepsilon_0 \bar{E}\,\hat{n} = \varepsilon_0 \bar{E}\,\hat{\theta} = \varepsilon_0\left[\frac{-10}{r\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)\operatorname{sen}\left(\frac{\pi}{6}\right)}\hat{\theta}\right]\hat{\theta} = -\frac{10\,\varepsilon_0\,\hat{\theta}}{r\ln\operatorname{tg}\left(\frac{\pi}{12}\right)\operatorname{sen}\left(\frac{\pi}{6}\right)} = \frac{15{,}1\,\varepsilon_0}{r}\hat{\theta}$$

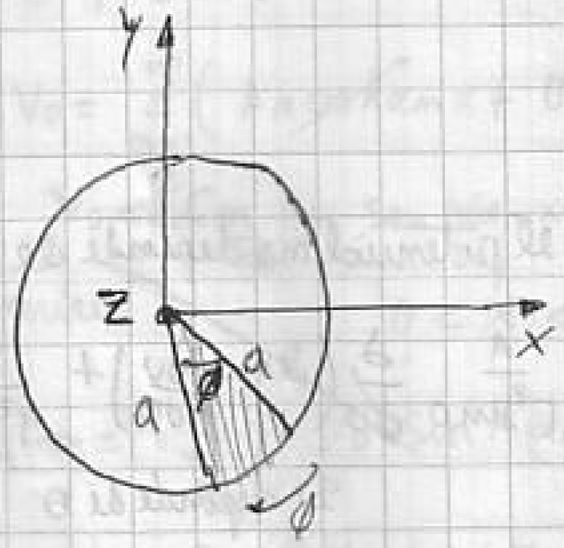
$$\sigma/_{\theta=\pi/2} = \varepsilon_0 E\,\hat{n} = \varepsilon_0 E(-\hat{\theta}) = \varepsilon_0\left[\frac{-10}{r\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)\operatorname{sen}\left(\frac{\pi}{2}\right)}\hat{\theta}\right](-\hat{\theta}) = \frac{10\,\varepsilon_0\,\hat{\theta}}{r\ln\left(\operatorname{tg}\left(\frac{\pi}{12}\right)\right)} = -\frac{7.57\,\varepsilon_0\hat{\theta}}{r}$$

?

3)




Solución:




En coordenadas cilíndricas: $\nabla^2 U = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial U}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial}{\partial \phi}\left(\frac{\partial U}{\partial \phi}\right) + \cancel{\frac{\partial^2 U}{\partial z^2}} = 0$

El potencial depende de $\rho$ y $\phi$ no de z:

$$U(\rho,\phi) = A_0 + B_0 \ln\rho + \sum_{m=1}^{\infty}(A_m \rho^m + B_m \rho^{-m})\cos m\phi + \sum_{m=1}^{\infty}(C_m \rho^m + D_m \rho^{-m})\,\mathrm{sen}\, m\phi \quad \dots (1)$$

Definiendo las condiciones de frontera:

$$U(\rho) = \begin{cases} \rho = a \;;\; U = 0 \quad (1^{ra}\,CF) \end{cases} \qquad U(\phi) = \begin{cases} U = V_0 \;;\; \phi = \phi \quad (2^{do}\,CF) \\ U = 0 \;;\; \phi = 0 \quad (3^{ra}\,CF) \end{cases}$$

Resolviendo:

$1^{ra}$ C.F:

$$\Rightarrow \underbrace{0 = A_0 + B_0 \ln a}_{} + \sum \overbrace{\left(A_1 a + \frac{B_1}{a}\right)}^{0}\cos\phi + \left(A_2 a^2 + \frac{B_2}{a^2}\right)\cos 2\phi + \dots + \left(A_m a^m + \frac{B_m}{a^m}\right)\cos m\phi$$
$$+ \left(C_1 a + \frac{D_1}{a}\right)\mathrm{sen}\,\phi + \left(C_2 a^2 + \frac{D_2}{a^2}\right)\mathrm{sen}\,2\phi + \dots \left(C_m a^m + \frac{D_m}{a^m}\right)\mathrm{sen}\, m\phi$$

~~$A_0 = 0,\ B_0 = 0$~~ $\quad A_1 a + \frac{B_1}{a} = 0 \dots\dots (2) \qquad A_0 + B_0 \ln a = 0 \dots (3)$

$C_m = D_m = 0 \;;\; m \geq 1 \quad A_m = B_m$

$2^{da}$ C.F:

$$\Rightarrow V_0 = A_0 + B_0 \ln\rho + \overbrace{\left(A_1 \rho + \frac{B_1}{\rho}\right)}^{0}\cos\phi + \left(A_2 \rho^2 + \frac{B_2}{\rho^2}\right)\cos 2\phi + \dots + \left(A_m a^m + \frac{B_m}{a^m}\right)\cos m\phi$$
$$\left(C_1 \rho + \frac{D_1}{\rho}\right)\mathrm{sen}\,\phi + \left(C_2 \rho^2 + \frac{D_2}{\rho^2}\right)\mathrm{sen}\,2\phi + \dots + \left(C_m a^m + \frac{D_m}{a^m}\right)\mathrm{sen}\, m\phi$$

$V_0 = A_0 + B_0 \ln\rho \dots (4)$

$3^{ra}$ C.F:

$$\Rightarrow 0 = A_0 + B_0 \ln a + \overbrace{\left(A_1 a + \frac{B_1}{a}\right)}^{0}\cos(0) + \left(A_2 a^2 + \frac{B_2}{a^2}\right)\cos(2\cdot 0) + \dots \left(A_m a^m + \frac{B_m}{a^m}\right)\cos(m\phi)$$
$$+ \left(C_1 a + \frac{D_1}{a}\right)\mathrm{sen}(0) + \left(C_2 a^2 + \frac{D_2}{a^2}\right)\mathrm{sen}(2\cdot 0) + \dots \left(C_m a^m + \frac{D_m}{a^m}\right)\mathrm{sen}(m\phi)$$

$\Rightarrow$ De (4) y (3): $\quad A_0 + B_0 \ln a = 0$, $\quad A_0 + B_0 \ln\rho = V_0 \quad \left\{ B_0 = \dfrac{V_0}{\ln\left(\frac{\rho}{a}\right)} \right.$

De (2) y $2^{da}$ C.F:

$A_1 a + \frac{B_1}{a} = 0$

$A_1 \rho + B_1 = 0$